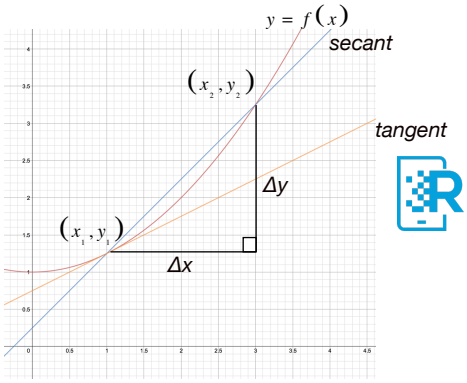
Mathematics 31

Sturgeon Composite High School

Derivatives

Slope Formula

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



Derivative Notations

$Lagrange - y'$ $\quad Euler - D_x y$

$Liebniz - \frac{dy}{dx}$ $\quad Newton - \dot{x}$

General Derivatives

Basic Rules

$$\frac{d}{dx}a = 0,\ a = constant$$

$$\frac{d}{dx}u = \frac{du}{dx} \qquad \frac{d}{dx}u = u'$$

$$\frac{d}{dx}a \cdot u = a \cdot \frac{du}{dx} \qquad \frac{d}{dx}a \cdot u = a \cdot u'$$

Power, Product & Quotient Rules

$$\frac{d}{dx}\left(u^n\right) = n \cdot u^{n-1} \cdot \frac{du}{dx} \qquad \frac{d}{dx}\left(u^n\right) = n \cdot u^{n-1} \cdot u'$$

$$\frac{d}{dx}(uv) = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx} \qquad \frac{d}{dx}(uv) = u'v + uv'$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx} \cdot v - u \cdot \frac{dv}{dx}}{v^2} \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

Chain Rule

$$\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx} \quad and \quad \frac{du}{dx} = \frac{du}{da} \cdot \frac{da}{db} \cdot \frac{db}{dc} \cdot \frac{dc}{dd} \cdot \cdots \cdot \frac{dr}{ds} \cdot \frac{ds}{dt} \cdot \frac{dt}{dx}$$

Exponential Derivatives

$$\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx} \qquad \frac{d}{dx}e^u = e^u \cdot u'$$

$$\frac{d}{dx}a^u = a^u \cdot \ln|a| \cdot \frac{du}{dx} \qquad \frac{d}{dx}a^u = a^u \cdot \ln|a| \cdot u'$$

Trigonometric Derivatives

$$\frac{d}{dx}\sin(u) = \cos(u) \cdot \frac{du}{dx} \qquad \frac{d}{dx}\cot(u) = -\csc^2(u)\frac{du}{dx}$$

$$\frac{d}{dx}\cos(u) = -\sin(u) \cdot \frac{du}{dx} \qquad \frac{d}{dx}\sec(u) = \sec(u)\tan(u)\frac{du}{dx}$$

$$\frac{d}{dx}\tan(u) = \sec^2(u) \cdot \frac{du}{dx} \qquad \frac{d}{dx}\csc(u) = -\csc(u)\cot(u)\frac{du}{dx}$$

Logarithmic Derivatives

$$\frac{d}{dx}\ln|u| = \frac{1}{u} \cdot \frac{du}{dx} \qquad \frac{d}{dx}\ln|u| = \frac{1}{u} \cdot u'$$

$$\frac{d}{dx}\log_a|u| = \frac{1}{u \cdot \ln|u|} \cdot \frac{du}{dx} \qquad \frac{d}{dx}\log_a|u| = \frac{1}{u \cdot \ln|u|} \cdot u'$$

Arc Trigonometric Derivatives

$$\frac{d}{dx}Sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \qquad \frac{d}{dx}Cot^{-1}(u) = \frac{-1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}Cos^{-1}(u) = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \qquad \frac{d}{dx}Sec^{-1}(u) = \frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}Tan^{-1}(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx} \qquad \frac{d}{dx}Csc^{-1}(u) = \frac{-1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

Integrals & Integration Techniques

Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

General Integral Techniques

Integration by Power Rule

$$\int x^n \, dx = \frac{1}{n+1} \cdot x^{n+1} + c$$

Integration by Substitution

$$\int u^n \, du = \frac{1}{n+1} \cdot u^{n+1} + c$$

Integration by Parts

$$\int u \, dv = uv - \int v \, du$$

Logarithmic Integration

$$\int \frac{1}{u} du = \ln|u| + c$$

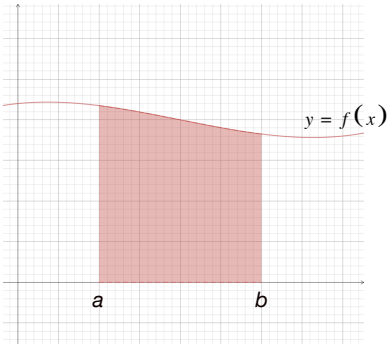
Exponetial Integration

$$\int e^u \, du = e^u + c$$

Partial Fractions

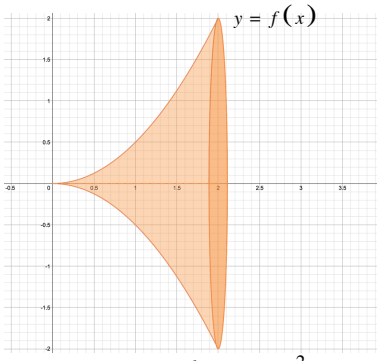
$$\int \frac{Ax+B}{(x-m)(x-n)} du = \int \frac{a}{(x-m)} du + \int \frac{b}{(x-n)} du + c$$

Area Under Curve



$$Area = \int_a^b f(x)dx$$

Volume of Rotation



$$Volume = \pi \int_a^b \left[f(x) \right]^2 dx$$

Trigonometric Substitions

Pythagorean Identities

$$\sin^2(x) = 1 - \cos^2(x)$$

$$\cos^2(x) = 1 - \sin^2(x)$$

$$\tan^2(x) = \sec^2(x) - 1$$

$$\csc^2(x) = 1 + \cot^2(x)$$

$$\sec^2(x) = 1 + \tan^2(x)$$

$$\cot^2(x) = \csc^2(x) - 1$$

Double Angle Identities

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\cos^2(x) = \frac{1}{2}\cos(2x) + \frac{1}{2}$$

$$\tan^2(x) = 1 - \frac{2\tan(x)}{\tan(2x)}$$

Special Integrals

$$\int \tan(u) du = \ln|\sec(u)| + c$$

$$\int \cot(u) du = \ln|\sin(u)| + c$$

$$\int \sec(u) du = \ln|\sec(u) + \tan(u)| + c$$

$$\int \csc(u) du = \ln|\csc(u) - \cot(u)| + c$$

$$\int \ln|u| \, du = u \ln|u| - u + c$$

Geometric Calculus Formulae

$$Area = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x, \quad x_i^* = midpoint\ of\ segment$$

$$Area = \int_a^b f(x) \, dx$$

$$Volume = \pi \int_a^b \left[f(x) \right]^2 dx$$

$$Length = \int_a^b \sqrt{1 + \left[f'(x) \right]^2} \, dx$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$